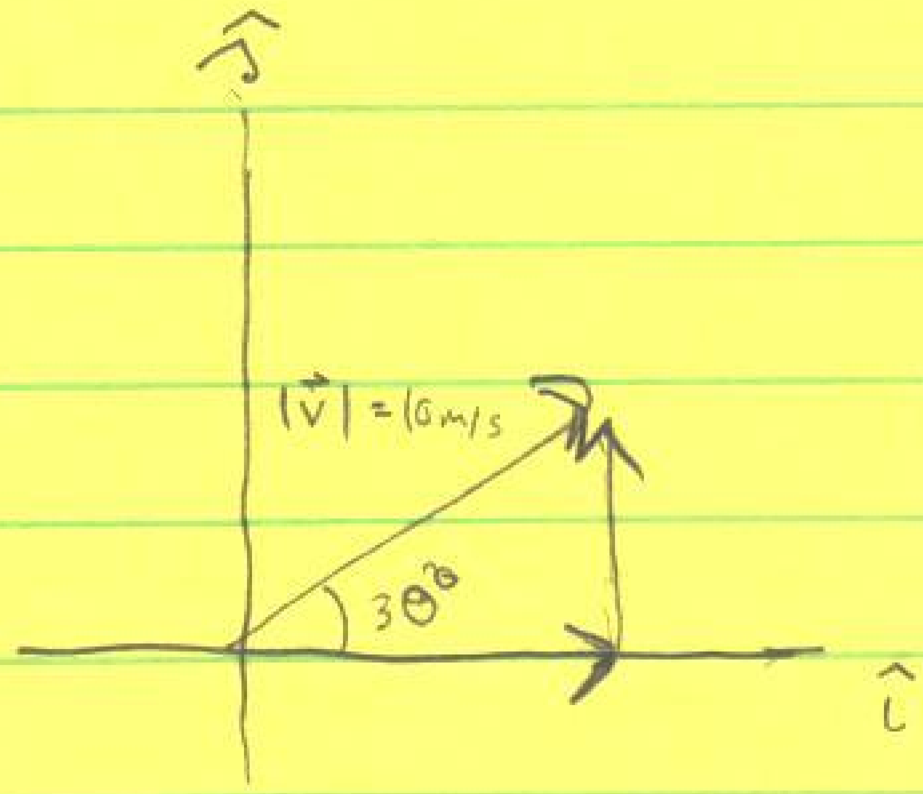
①

<u>Last time</u>

<u>Vectors</u> - Joe sets off 30° NE with constant speed 10m/s. What is his velocity in $\hat{i}, \hat{j}$ notation



- His initial

$$\vec{V}_{joe} = 10\,m/s \cos 30^\circ\,\hat{i} + 10\,m/s \sin 30^\circ\,\hat{j}$$

$$= 8.6\,m/s\,\hat{i} + 5\,m/s\,\hat{j}$$

Sue sets of with:

$$\vec{V}_{Sue} = -17.32\,\hat{i} + 10\,m/s\,\hat{j}$$ which remains constant

<u>Prblm</u>

① Draw this vector, find the speed of Sue, find the angle relative to some one heading east

$$\text{speed} = \sqrt{(17.32)^2 + 10^2} \approx 20\,m/s$$

$$\sin\alpha = \frac{10\,m/s}{20\,m/s} \Rightarrow \alpha = 30^\circ, \Rightarrow \theta = 180^\circ - 30^\circ = 15$$

②

Problem

Relative to joe, what is sues velocity?

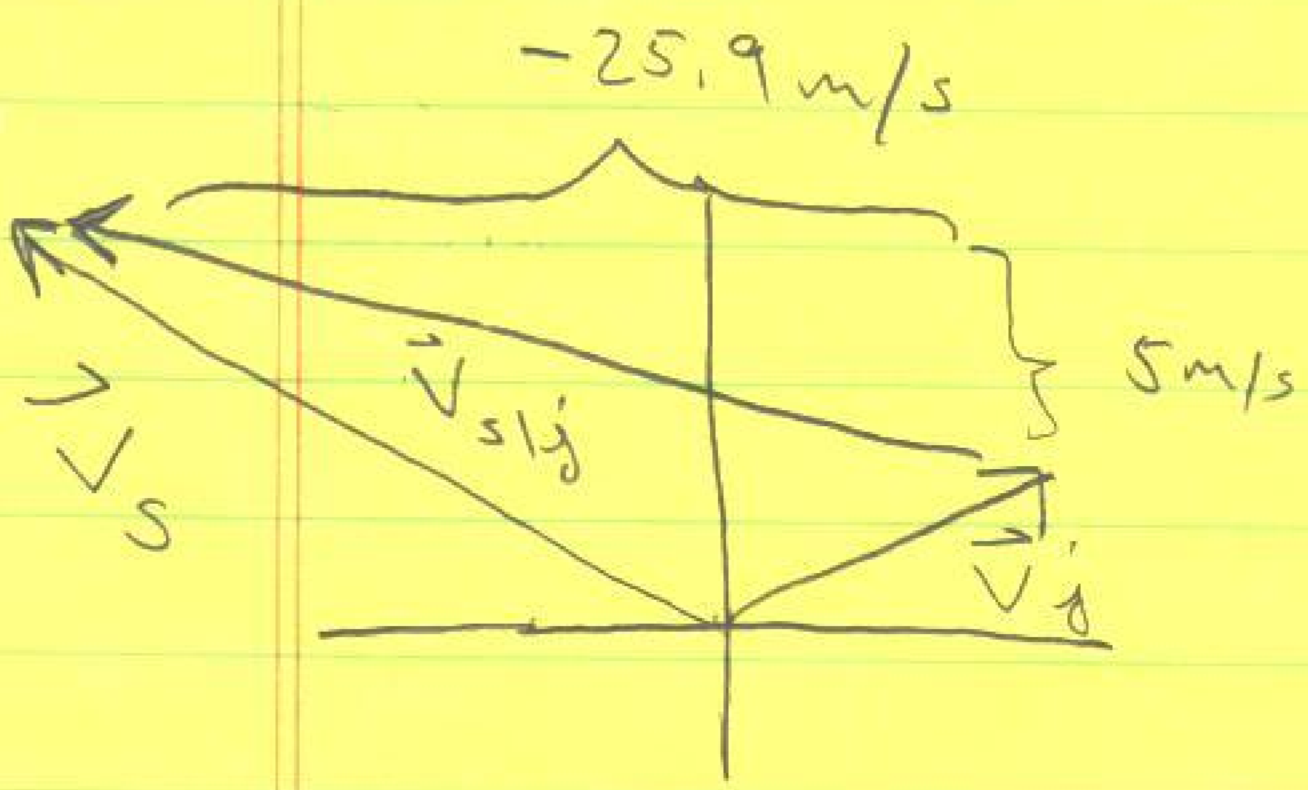
Solution $\vec{V}_{sue} = \vec{V}_{joe} + \vec{V}_{sue|joe}$

$$\vec{V}_{sue|joe} = \vec{V}_{sue} - \vec{V}_{joe}$$

$$\vec{V}_{sue|joe} = \begin{pmatrix} -17.3 \text{ m/s} \\ 10 \text{ m/s} \end{pmatrix} - \begin{pmatrix} 8.6 \text{ m/s} \\ 5 \text{ m/s} \end{pmatrix}$$

$$= \begin{pmatrix} -25.9 \text{ m/s} \\ 5 \text{ m/s} \end{pmatrix}$$

Graphically



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summary:

$$\vec{V}_{joe} = \begin{pmatrix} 8.6\ m/s \\ 5\ m/s \end{pmatrix} \qquad \vec{V}_{sue} = \begin{pmatrix} -17.3\ m/s \\ 10\ m/s \end{pmatrix}$$

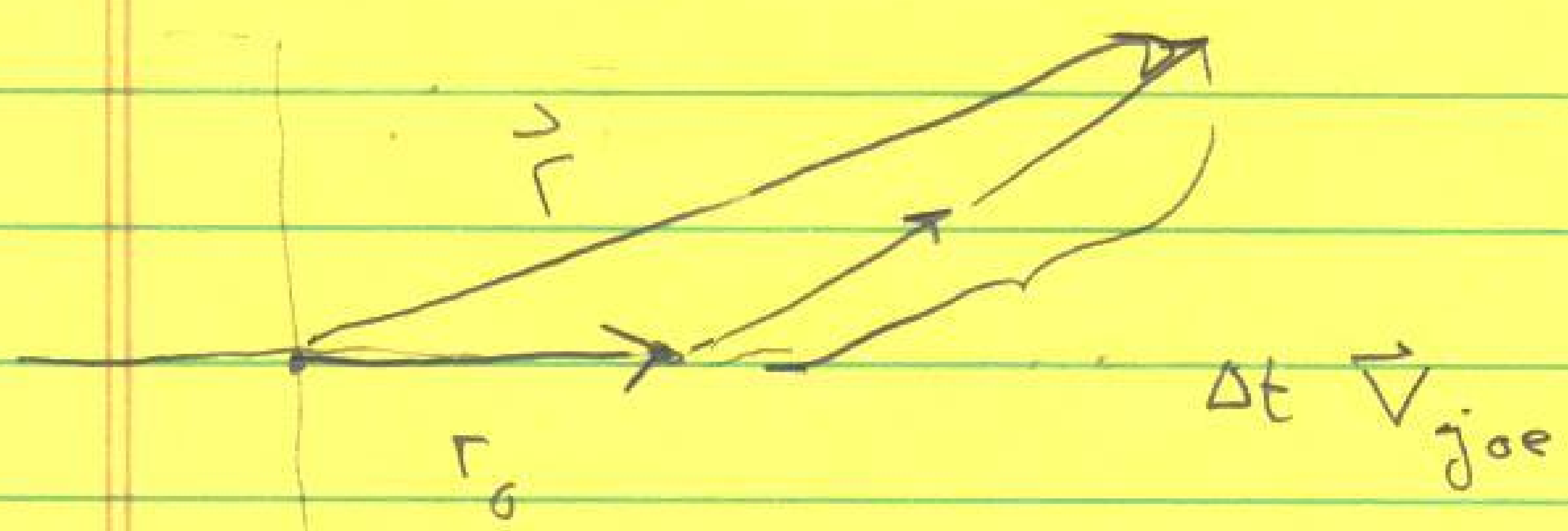
Problem

Suppose joe starts 10 m east of sue (who we take at the origin) what are the positions of joe and sue after 2s, what is the distance between them

General

Joe and sue move with constant velocity:

$$\vec{r}(t) = \vec{r}_0 + \vec{v}\ t \qquad \vec{v} = \frac{d\vec{x}}{dt}$$



3a)

joes initial position

$\vec{r}_0 = \begin{pmatrix} 10m \\ 0 \end{pmatrix}$ $\quad\quad$ $\vec{v} = \begin{pmatrix} 8.6 m/s \\ 5 m/s \end{pmatrix}$

$\vec{r}_j = \vec{r}_0 + \vec{v}\, t$ $\quad\quad$ $\vec{v} = 8.6 m/s\, \hat{i} + 5 m/s\, \hat{j}$

$\vec{r}_j = \begin{pmatrix} 10 m \\ 0 \end{pmatrix} + \begin{pmatrix} 8.6 m/s \\ 5 m/s \end{pmatrix} \cdot 2s$

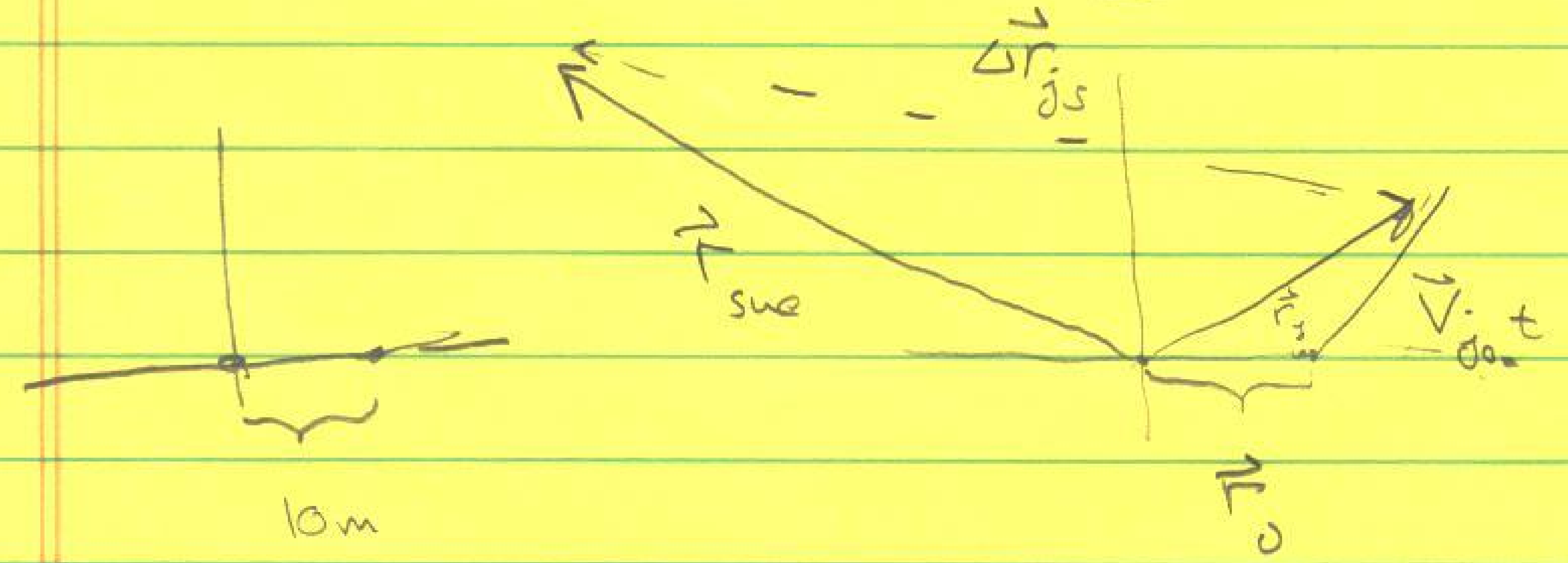
$\vec{r}_{joe} = \begin{pmatrix} 10m + 17.2m \\ 0 m + 10 m \end{pmatrix} = \begin{pmatrix} 27.2m \\ 10m \end{pmatrix}$

$\vec{r}_{sue} = \vec{r}_{0,sue} + \vec{v}_{sue}\, t$

$\vec{r}_{sue} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -17.3 m/s \\ 10 m/s \end{pmatrix} \cdot 2s$

$\vec{r}_{sue} = \begin{pmatrix} -34.6m \\ 20m \end{pmatrix}$

$\underline{t=0}$ $\quad\quad\quad\quad$ $\underline{t=2}$



(3a)

$$\Delta r_{joe \to sue} = \vec{r}_{sue} - \vec{r}_{joe}$$

or

$$\vec{r}_{joe} + \Delta\vec{r}_{joe \to sue} = \vec{r}_{sue}$$

$$\Delta\vec{r}_{j \to s} = \begin{pmatrix} -34.6m \\ 20m \end{pmatrix} - \begin{pmatrix} 27.2m \\ 10m \end{pmatrix} = \begin{pmatrix} -61.8\,m \\ 10\,m \end{pmatrix}$$

$$\Delta r_{j \to s} = -61.8\,m\,\hat{i} + 10m\,\hat{j}$$

Distance

Between

them $= |\Delta r_{j \to s}| = \sqrt{(-61.8)^2 + 10^2} \simeq 62.6\,m$

angel = ? $\tan\phi = \dfrac{10m}{-61.8\,m} \Rightarrow \phi = 180^\circ - 9.2^\circ$

Relative Velocity

$$\vec{r}_j = \vec{r}_{oj} + \vec{v}_j t \qquad \vec{r}_s = \vec{r}_{os} + \vec{v}_s t$$

$$\vec{r}_j = \vec{r}_{oj} + \vec{v}_j t$$

$$\Delta\vec{r}_{js} = (\vec{r}_s - \vec{r}_j) = (\vec{r}_{os} - \vec{r}_{oj}) + (\vec{v}_s - \vec{v}_j)\,t$$

$$\Delta r_{js} = (\Delta r)_{ojs} + (v_{s|j})\,t$$

3c

So

$$\left(\vec{\Delta r}_{js}\right)_0 = \begin{pmatrix} -10\,m \\ 0 \end{pmatrix}$$

t = 0

$$\Delta r_{js} = \begin{pmatrix} -10\,m \\ 0 \end{pmatrix} + \overbrace{\begin{pmatrix} -25.9 \ m/s \\ 5 \ m/s \end{pmatrix}}^{\vec{V}_{sue} - \vec{v}_{joe}} \cdot 2$$

$$r_{js} = \begin{matrix} -61.8\ m/s \\ 10\ m/s \end{matrix}$$

Then what about constant acceleration

$$\vec{V} = \vec{V}_0 + \vec{a}\,\Delta t \longrightarrow \frac{\Delta \vec{V}}{\Delta t} = \vec{a}$$

$$\vec{r}(t) = \vec{r}_0 + \vec{V}_0 t + \frac{1}{2}\vec{a}\,t^2$$

This is actually two equations each

$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} V_{0x} \\ V_{0y} \end{pmatrix} + \begin{pmatrix} a_x \\ a_y \end{pmatrix} t$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} V_{0x} \\ V_{0y} \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} a_x \\ a_y \end{pmatrix} t^2$$

→ The equations for x and y are separate

Ex, A log jumper leaves the ground at an angle of 20° with speed 11 m/s

1. How long is he in the air?
2. How far does he jump?

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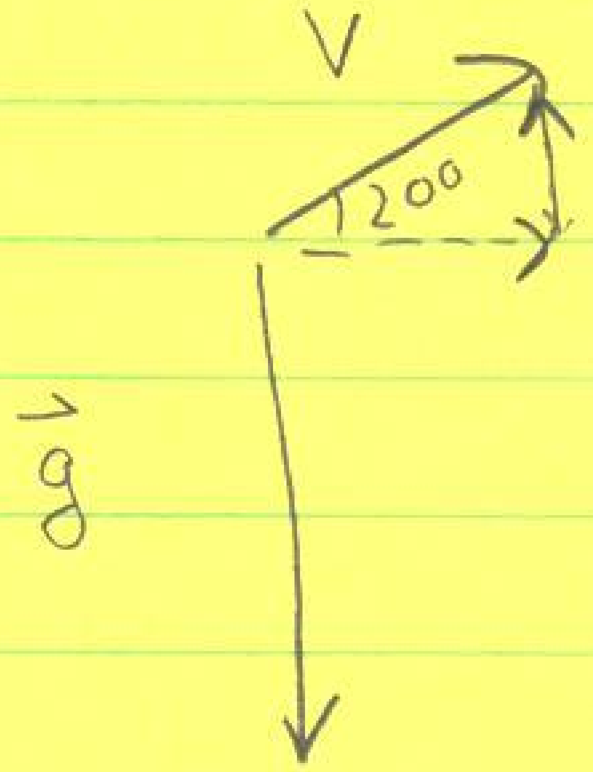
Solution

1 Sketch



$\vec{V} = 11 \cos 20$

2 Resolve velocity vector into x and y

3 Resolve the acceleration vector into x + y

4 Solve x and y separately

$$\vec{V}_0 = (11 \cos 20^\circ)\,\hat{i} + (11 \sin 20^\circ)\,\hat{j} \rightarrow \begin{pmatrix} 10.33 \text{ m} \\ 3.7 \text{ m} \end{pmatrix}$$

$$\vec{a} = -g\,\hat{j} \rightarrow \begin{pmatrix} 0 \\ -g \end{pmatrix}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\,t^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \overbrace{V_{0x}}^{10.33 \text{ m/s}} \\ 3.7 \text{ m/s} \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$$

$$x(t) = \overbrace{V_{0x}}^{10.33 \text{ m/s}}\, t$$ ← his x increases linea[rly] he is not accelerat[ing]

$$y(t) = \overbrace{V_{0y}}^{3.7 \text{ m/s}}\, t - \frac{1}{2}(g)\,t^2$$ ← one dimensional motion

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How long was he in the air

$\underbrace{\text{How long}}_{t_*}$ $\underbrace{\text{was he in the air}}_{\text{when } y=0}$

$y(t_*) = 0$

$3.7\,m/s\,t - \frac{1}{2}(9.8\,m/s^2)\,t^2 = 0 \qquad t_* = 0.75\,s$

How far did he go?

$\underbrace{\text{How far}}_{x_*}$ – increases until he hits the ground, $t = t_*$

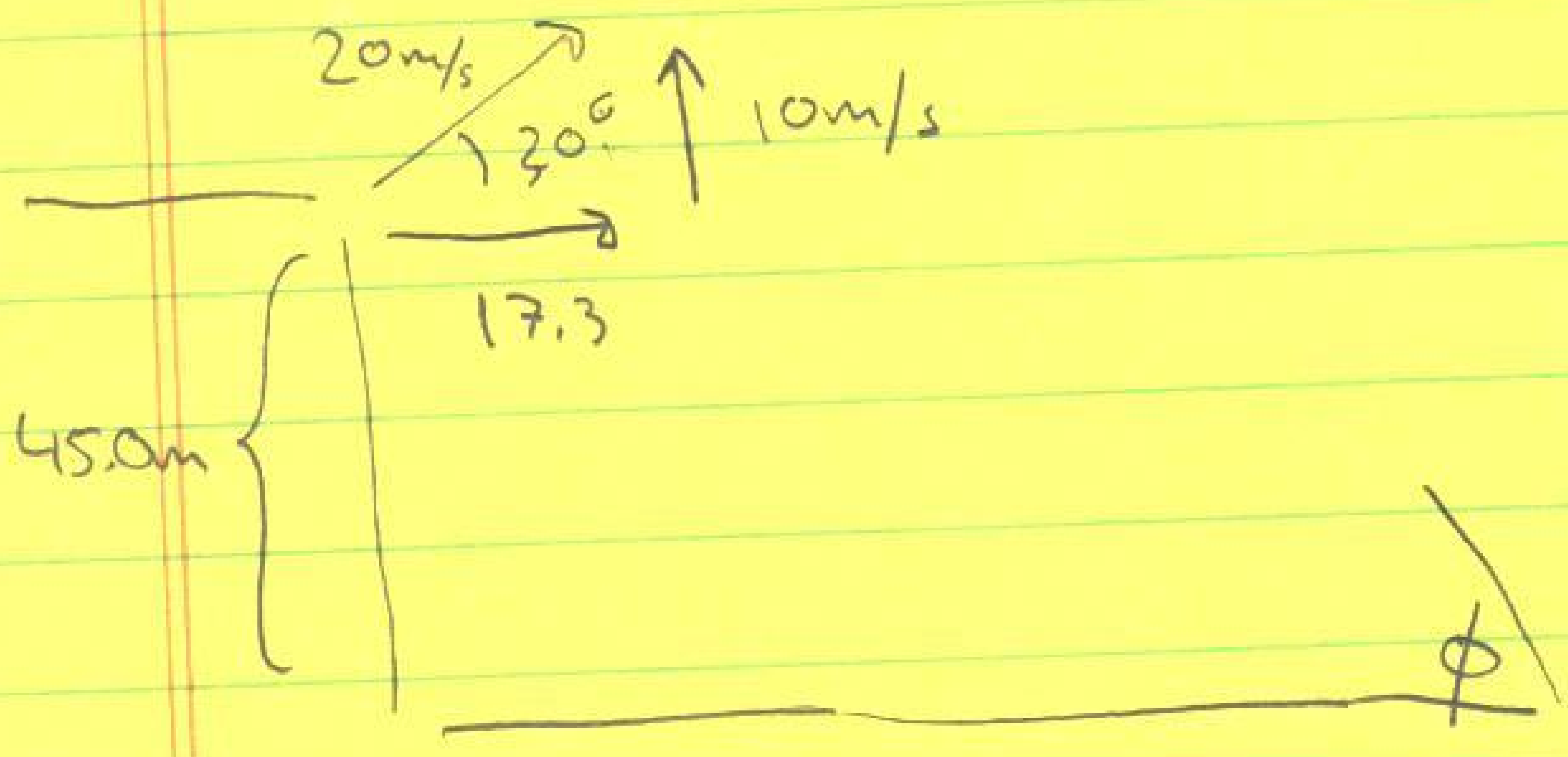
$x(t) = V_{0x}\,t$

$x_* = V_{0x}\,t_* = (10.33\,m/s)(0.75\,s)$

$\boxed{x_* = 7.74\,m}$

Demo

(7)



(1) When does it hit the ground? $t = 4.22s$

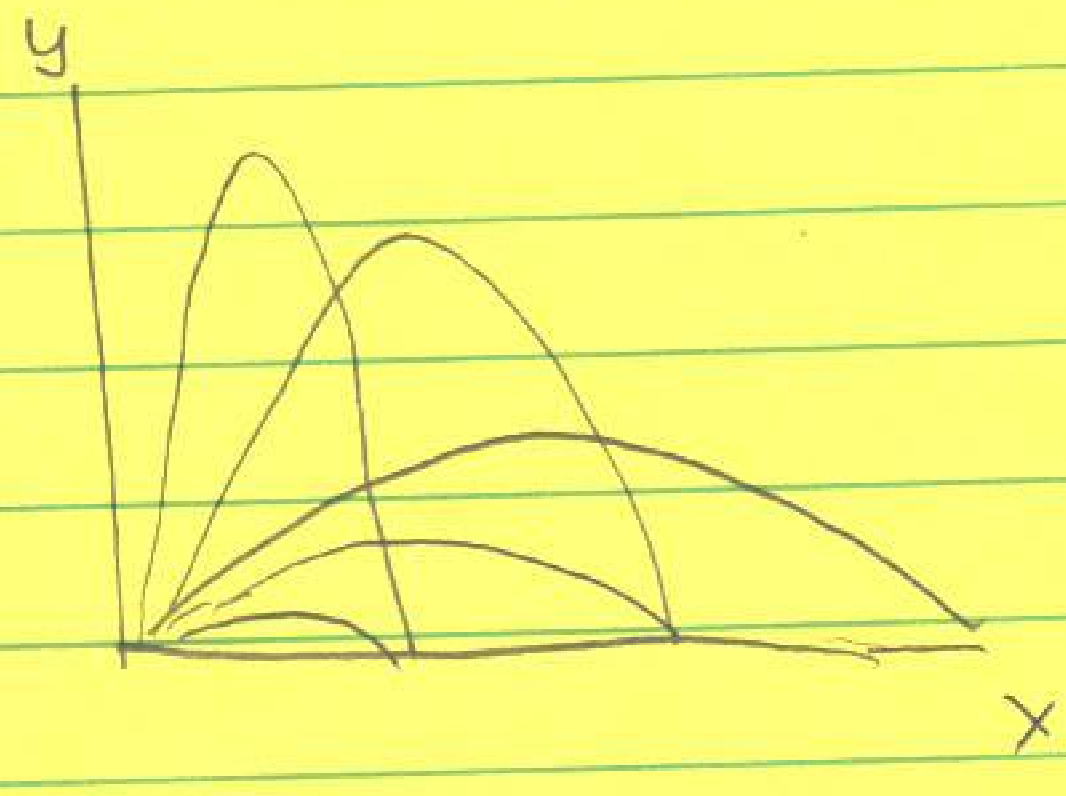
(2) How far does it travel? 73.0m

(3) How fast is it travelling when it hits the ground, whats the angle ϕ?

$V_{yf} = -31.4 m/s$ $\sqrt{|v|^2} = 35.9$

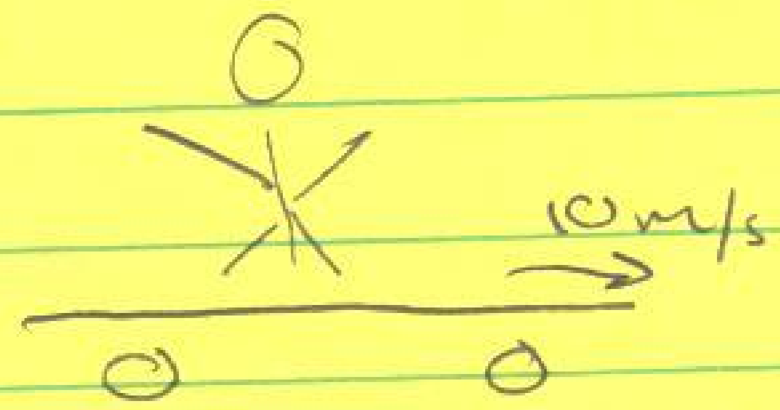
$V_{xf} = 17.3 m/s$ $\phi = 61^0$

(8) (1) Conceptual Question

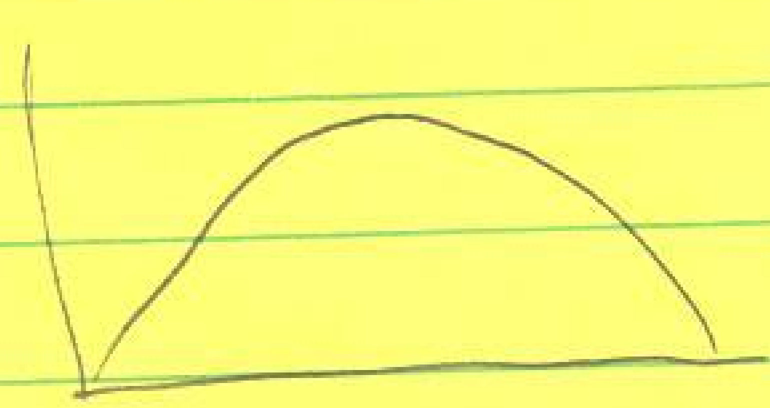


Rank trajectories, shortest time of flight to longest

(2) You are travelling on a cart. which way should you throw a tennis ball so it comes back to your hand

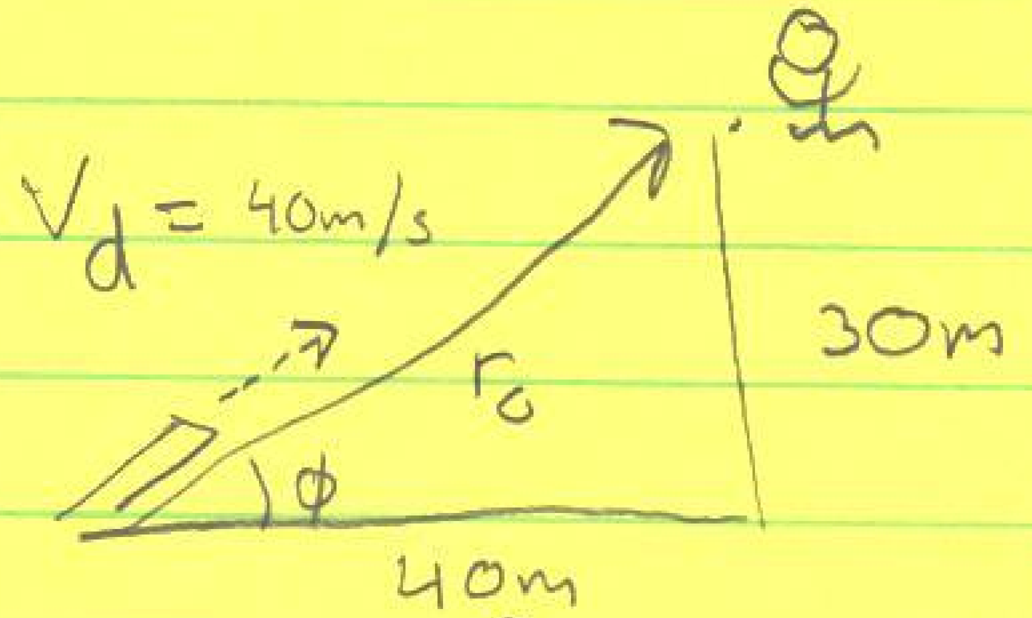


(3) Plot the velocity and acceleration vecs. for this trajectary



(9)

Monkey and Bannanas



$$\vec{x}_M = \vec{r}_0 + \cancel{\vec{v}_0} t - \frac{1}{2}\vec{g}\,t^2$$

$$\vec{x}_d = \vec{v}_0\, t - \frac{1}{2}\vec{g}\,t^2$$

$$\vec{x}_M = \vec{x}_d$$

$$\vec{r}_0 - \frac{1}{2}\vec{g}\,t^2 = \vec{v}_0\, t - \frac{1}{2}\vec{g}\,t^2$$

$$\vec{r}_0 = \vec{v}_0 t \qquad \vec{v}_0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} v_{ox}\, t \\ v_{oy}\, t \end{pmatrix} \Longrightarrow \tan\phi = \frac{v_{oy}}{v_{ox}} = \frac{y}{x}$$

When and where does the dart strike the monkey

(1)

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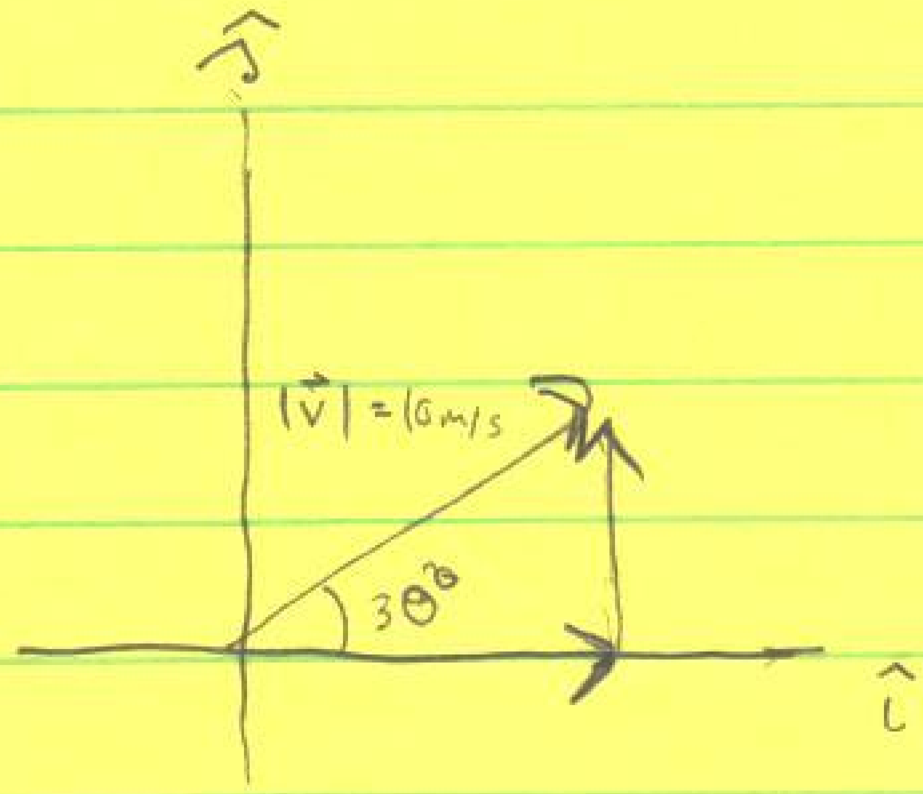

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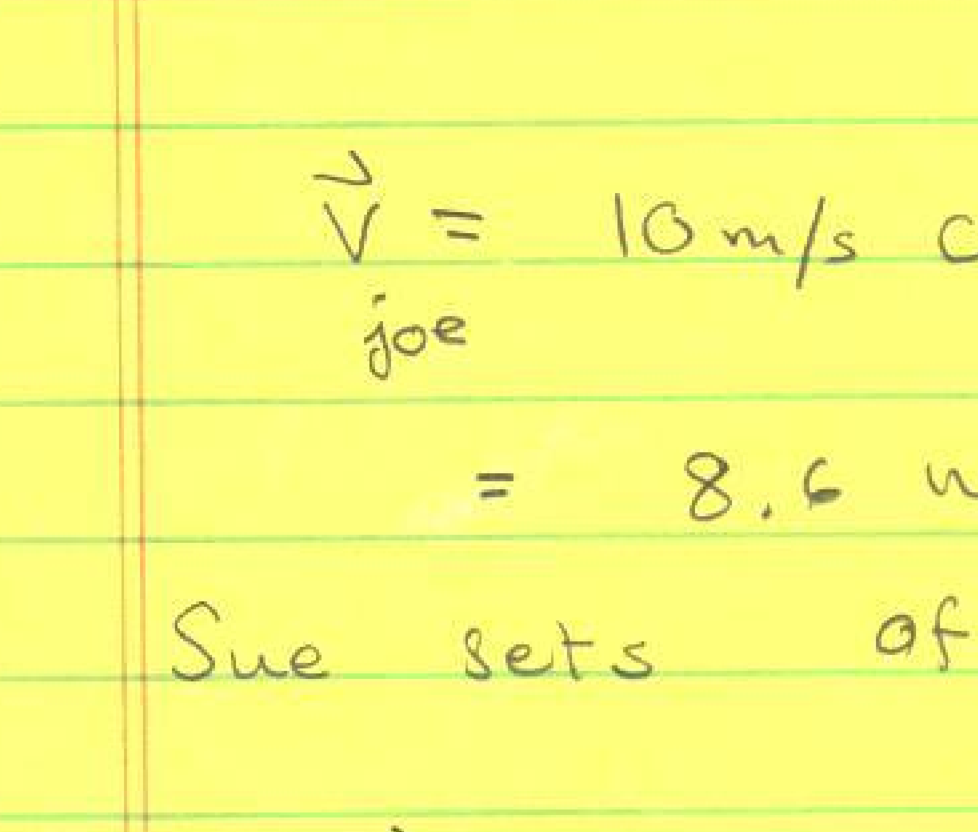

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